OPTIMIZATION OF ATMOSPHERIC MODELS

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SUMMARY

Applying variational methods to a mathematical model of the atmosphere an entirely new type of equations for forecasting atmospheric parameters is derived. The method also defines vertical eigenfunctions to the model. In a simplified case some of the eigenfunctions are compared with empirically obtained data and conclusions are drawn regarding the validity of some of the approximations in the mathematical model.

SAMMANFATTNING

Genom en tillämpning av variationskalkyl på de ekvationer, som definierar en matematisk modell av atmosfären härleds en helt ny typ av ekvationer för prognos av modellens parametrar. Metoden definierar också vertikala egenfunktioner till modellen. I ett förenklat fall jämförs några av dessa egenfunktioner med empiriska data och resultatet utnyttjas för att bedöma riktigheten i några av de approximationer, som gjorts i den matematiska modellen.

1. INTRODUCTION

In the terminology of numerical weather prediction a forecasting parameter is a function of x,y and t only (or λ, ϕ and t) substituting the dependent variable we wish to forecast by means of non-linear predictive equations. A compact and very general form for the transformation from a time and space dependent variable to a parameter may be obtained by a series expansion. Taking for instance the stream function $\psi(x,y,t,p)$ as dependent variable, the expansion

$$\psi(x,y,t,p) = \Sigma\ \alpha_n(x,y,t)\ F_n(p) \tag{1}$$

will determine the forecasting parameters $\alpha_n(x,y,t)$ as soon as the functions $F_n(p)$ are given. In integrated models these will be suitably chosen continuous weighting functions, while, taking them as delta functions, we instead obtain parameters representing conditions at prescribed levels.

The form of the series expansion (1) shows that what we in fact are doing, when carrying out the transformation, is to utilize the method of variable separation, so common and efficient in solving linear partial differential equations. However, when application is made to non-linear partial differential equations, the purpose of the method is not very clear and it is therefore of interest to make a comparison between these two very different situations.

In the case of a linear equation, the expansion (1) is not arbitrary. Instead, it is defined by the requirement that each separate term in the series should exactly satisfy the equation and furthermore that the functions $F_n(p)$, in this case the eigenfunctions of the problem, should satisfy certain boundary conditions. The rate of convergence of the series is here not of particular

interest, it is determined partly by initial conditions, i.e. the extent to which various modes are excited initially, and partly by damping or diffusing terms in the equation which generally result in high frequency and high wavenumber modes vanishing rapidly.

In the non-linear case particular solutions that exactly satisfy the equation do not exist, except in a few special cases which here are of no interest. Therefore, in our usual approach to a non-linear problem, the functions $F_n(p)$ in (1) remain undetermined and arbitrary and one may here choose any complete orthogonal set provided boundary conditions can be satisfied. For reasons of formal simplicity Fourier functions are often selected but again it should be noted that these are not in any specific way related to the equation. Due to the non-linearity energy transfer will take place between possible modes, and the rate of convergence in the series- or in other terms the energy spectrum will depend on the choice of the orthogonal set. Obviously one may here by misfortune make a poor choice and end up with a very slow convergence. In the application discussed here this is equivalent to a need for many parameters before a satisfactory approximation of the dependent variable is obtained.

In order to economise on the number of parameters and, by this, also on computer time and capacity we obviously need a method by which a best choice or a consistent derivation of the expanding functions $F_n(p)$ can be made. With this purpose in mind it is natural to return to the principle of eigenfunctions but, in view of the non-linearity, to generalize and extend their definintion. Retaining the condition that they should satisfy proper boundary conditions we may now define them by the relaxed requirement that they should satisfy the equation not exactly but instead as well as possible. This means that the determination of eigenfunctions transforms into a well-defined variational problem. The definition of

eigenfunctions to linear equations will remain unchanged as a special, degenerate case, since here the minimum will be exactly zero.

2. Some properties of generalized eigenfunctions

In order to clarify the ideas discussed above we shall consider a dependent variable $u(x,y,t)$ the evolution of which is governed by the non-linear equation

$$M(u) = 0 \tag{2.1}$$

M being a non-linear operator. Equation (2.1) is supposed to be valid in the area S and u should satisfy given boundary conditions along the boundary L. Furthermore u is supposed to satisfy some initial conditions.

For $u(x,y,t)$ we now introduce an approximation denoted by $v(x,y,t)$. We may require v to be of the form

$$v(x,y,t) = \alpha(t)\, f(x,y) \tag{2.2}$$

but other restrictions are also possible. With this limitation, v can no longer satisfy equation (2.1) and we therefore have

$$M(v) = R(x,y,t) \tag{2.3}$$

where the values of the residual R depend on the approximation or restriction made in v. Within these limitations we now wish to make the best possible choice and we therefore introduce the condition that R should be minimized in the least square sense when integrated over S and over the time interval $(0,T)$. The variational problem is thus

$$\iint_{ST} R^2 \, ds\, dt = \text{minimum}$$

or in view of (2.3)

$$\iint_{S\,T} R\ \delta\ M(v)\,ds\,dt = 0 \qquad (2.4)$$

Since variations by definition are small, this equation will naturally be linearized in terms of the variation δv. If $M(u)$ for instance contains the term $u\ \partial u/\partial x$ we shall have

$$\iint_{S\,T} R\ \delta\ (v\frac{\partial v}{\partial x})\ ds\,dt = \iint_{S\,T} (R\frac{\partial v}{\partial x}\ \delta v + R\,v\,\delta\frac{\partial v}{\partial x})\ ds\,dt$$

By partial integration the second term may be transformed so that the final result becomes

$$\iint_{S\,T} R\,\delta\ (v\frac{\partial v}{\partial x})\ ds\,dt = \iint_{S\,T} (R\frac{\partial v}{\partial x} - \frac{\partial Rv}{\partial x})\,\delta\,v\,ds\,dt +$$

$$+ \iint_{Y\,T} \Big[R\,v\,\delta\ v\Big]_{x_1}^{x_2}\ dy\,dt$$

Here, if v is supposed to be zero at the boundary we have $\delta v = 0$ at x_1 and x_2 and the second integral vanishes. If this condition is not imposed on v the second integral, possibly together with other partially integrated terms will provide natural boundary conditions on v. In any case the first integral above will separate out and together with other terms multiplying the variation δv in S and T give an equation of the form

$$\iint_{S\,T} M'(R,v)\ \delta v\,ds\,dt = 0 \qquad (2.5)$$

where M' is a new operator on v and R, involving non-linear partial derivatives. In particular, if M includes partial time derivatives M' will also do so and is thus a prognostic operator.

In (2.5) we may now consider the case when the restric-

tion on $v(x,y,t)$ is one of variable separation according to (2.2). We thus have

$$\delta v = \alpha(t)\,\delta f(x,y) + f(x,y)\,\delta\alpha(t) \qquad (2.6)$$

where δf and $\delta\alpha$ naturally are independent. Equation (2.5) therefore separates into the following two

$$\int_T M'(R,v)\,\alpha(t)\,dt = 0 \qquad (2.7)$$

and

$$\int_S M'(R,v)\,f(x,y)\,ds = 0 \qquad (2.8)$$

Comparing now (2.5), (2.7) and (2.8) it is seen that if v is permitted an unrestricted variation δv the equation

$$M'(R,v) = 0 \qquad (2.9)$$

will be a correct prognostic equation for v. If on the other hand the variation δv is restricted according to (2.6) then (2.9) is no longer valid. However, in this case it is seen from (2.7) and (2.8) that the operator $M'(R,v)$ is orthogonal to $\alpha(t)$ and $f(x,y)$ and this implies that if (2.9) is still used as a predictive equation for v, then the error field, created at each instant will be orthogonal to the approximated variable.

In practical application (2.9) will not be used for forecasting purpose. Instead equation (2.7), being integrated over t will provide a non-linear partial differential equation which together with given and natural boundary conditions will determine $f(x,y)$ as a solution. The coefficients in this equation will depend on statistics of $\alpha(t)$ and its time derivatives. In a similar way equation (2.8), being integrated over S, will provide the necessary predictive equation for $\alpha(t)$, in which

the coefficients will depend on surface statistics of $f(x,y)$ and its derivatives.

If we multiply equation (2.8) by $\alpha(t)$ and integrate over time we obtain a relation between these two types of statistics. This corresponds to the usual relation between eigenvalues that one finds in linear problems. It follows that by this method different modes can be determined where, in each separate mode, an interdependence exists between time scale and horizontal scale.

Considering the predictive equation (2.8) it is easily shown by partial integration that if the operator M of (2.1) includes the time derivative $\partial/\partial t$, then the operator M' will have $\partial^2/\partial t^2$ as highest order time derivative. This means that the forecast equation for α is very different from the initial forecast equation (2.1). In other words, in order for an approximation to satisfy a prognostic equation as well as possible its evolution in time should not be calculated by introduction in this same equation but instead in a different equation of higher order. It should be noticed that this also holds true if in (2.2) we prescribe the function $f(x,y)$ and thus only deal with a variation of α. The resulting forecast equation for $\alpha(+)$ will still be equation (2.8).

The procedure given above may now be extended to include more terms in the approximation of $u(x,y,t)$. Taking for instance

$$v = \alpha_1(t)\ f_1(x,y) + \alpha_2(t)\ f_2(x,y) \qquad (2.10)$$

we have

$$\delta v = \alpha_1 \delta\ f_1 + f_1\ \delta\ \alpha_1 + \alpha_2 \delta\ f_2 + f_2\ \delta\ \alpha_2$$

and since these variations are independent, equations (2.7) and (2.8) transform into

$$\text{a)} \int_T M'(R,v)\,\alpha_1\,dt = 0 \qquad \text{b)} \int_S M'(R,v)\,f_1\,ds = 0$$

$$\text{c)} \int_T M'(R,v)\,\alpha_2\,dt = 0 \qquad \text{d)} \int M'(R,v)\,f_2\,ds = 0 \tag{2.11}$$

which shows that the orthogonality considerations made before still hold. Since it is not possible in (2.10) and (2.11) to discriminate between the first and the second term, an extra condition has to be made. Various possibilities may here exist but the simpliest is probably to consider $f_1(x,y)$ as prescribed, determined instead from (2.7) and thus excluding equation a) from (2.11). Equation c) in (2.11) will then determine f_2 while forecast equations for α_1 and α_2 are obtained from equations b) and d). This procedure may then be extended to any required number of terms.

3. The model atmosphere

In order now to apply the method to the construction of an atmospheric model we shall here consider the simplest possible case, starting from the vorticity and adiabatic equations in the following, approximate form

$$\frac{\partial \nabla^2\psi}{\partial t} + \beta\,\frac{\partial\psi}{\partial x} + J(\psi,\nabla^2\psi) - f_0\,\frac{\partial\omega}{\partial p} = 0 \tag{3.1}$$

and

$$\frac{\partial^2\phi}{\partial t\partial p} + J(\psi,\,\frac{\partial\phi}{\partial p}) + \omega\sigma_0 = 0 \tag{3.2}$$

Elimination of ω and introduction of the geostrophic approximation $f_0\psi = \phi$ gives

$$\frac{\partial \nabla^2\psi}{\partial t} + \beta\,\frac{\partial\psi}{\partial x} + J(\psi\;\nabla^2\psi) + f_0^2\,\frac{\partial}{\partial p}\,\frac{1}{\sigma_0}\left[\frac{\partial^2\psi}{\partial t\partial p} + J(\psi,\,\frac{\partial\psi}{\partial p})\right] = 0 \tag{3.3}$$

We now introduce the following approximation for the stream-functions

$$\psi(x,y,t,p) \simeq \alpha(x,y,t)\, F(p) \tag{3.4}$$

and obtain from (3.3) the relation

$$\left(\frac{\partial \nabla^2 \alpha}{\partial t} + \beta\, \frac{\partial \alpha}{\partial x}\right) F + J(\alpha, \nabla^2 \alpha) F^2 + f_0^2\, \frac{\partial \alpha}{\partial t}\, \frac{d}{dp}\left(\frac{1}{\sigma_0}\, \frac{dF}{dp}\right) = \tag{3.5}$$

$$= R(x,y,t,p)$$

where R is the residual due to the restricted form of the approximation. For the same reason and due to the geostrophic approximation the Jacobian from the adiabatic equation will take the form

$$f_0^2\; J(\alpha,\alpha) \frac{d}{dp}\left(\frac{F}{\sigma}\, \frac{dF}{dp}\right)$$

and thus vanish. It should however already here be pointed out that friction would make the Jacobian non-vanishing and that the term therefore may be of importance. We shall return to this question later on.

In order now to have a boundary condition at the surface of the earth we introduce for the geopotential ϕ an approximation consistent with (3.4) or

$$\phi(x,y,t,p) = F_0(p) + f_0 \alpha(x,y,t) F(p)$$

where $F_0(p)$ represents a standard atmosphere.

Taking here $\phi = 0$ and differentiating the expression on the right hand side with respect to time, we obtain

$$\frac{dF_0}{dp}\, \frac{dp}{dt} + f_0 \alpha \frac{dF}{dp}\, \frac{dp}{dt} + f_0 \frac{d\alpha}{dt} F = 0$$

or

$$f_0 \frac{d\alpha}{dt}\, F + \omega\left(\frac{dF_0}{dp} + f_0 \alpha\, \frac{dF}{dp}\right) = 0 \tag{3.6}$$

Introducing now the same approximation into the adiabatic equation, it may be written

$$f_0 \frac{d\alpha}{dt} \frac{dF}{dp} + \omega\sigma_0 = 0 \qquad (3.7)$$

noting that here the operator d/dt only includes horizontal derivatives. In (3.6) we now neglect the second term in the parenthesis multiplying ω and obtain from a combination of (3.6) and (3.7) the boundary condition at $p = p_s$

$$S_0 \frac{dF}{dp} - \sigma_0 F = 0$$

with $S_0 = dF_0/dp$, the negative value of standard specific volume at surface pressure.

With regard to the upper boundary at $p = 0$ the only condition required is that F should remain finite since this point is singular due to $1/\sigma_0$ being zero.

4. Variation of α

With the form given in (3.4) for the approximation of ψ, the variational problem directly separates into the two equations

$$\iiint_{STP} R\, \delta_\alpha R\, ds\, dt\, dp = 0$$

and (4.1)

$$\iiint_{STP} R\, \delta_F R\, ds\, dt\, dp = 0$$

with R given by (3.5).

Here the variational symbol δ_α indicates that only α and not F is to be varied and vice versa for δ_F. S is the horizontal area of integration with $ds = dx\,dy$. The boundary of S will be denoted L with dL a line element on this boundary. Furthermore T indicates the interval of time over which the forecast equation is to be valid and P the vertical pressure interval, here taken to be $(0,p_s)$ with p_s equal to surface pressure.

In order to have a more compact form of the equation we also here introduce the notation

$$F^* = f_0^2 \frac{d}{dp} \left(\frac{1}{\sigma_0} \frac{dF}{dp}\right) \tag{4.2}$$

The variational equation to be considered here is first

$$\iiint\limits_{S\,T\,P} R\,\delta_\alpha \left[\left(\frac{\partial \nabla^2 \alpha}{\partial t} + \beta \frac{\partial \alpha}{\partial x}\right) F + J(\alpha, \nabla^2 \alpha) F^2 + \right.$$
$$\left. + \frac{\partial \alpha}{\partial t} F^* \right] ds\,dt\,dp = 0 \tag{4.3}$$

where partial integrations have to be carried out in order to obtain integrands which explicity contain δ_α or its lowest possible derivatives as a factor.

The manipulations are rather long and tedious and details of them have therefore been collected in appendix A. For the discussion it is sufficient here to give the resulting full equation. After separation into different and independent variations of α or its derivatives we obtain the following integrals, which all must vanish separately.

$$\iiint\limits_{STP} \left[F\frac{\partial \alpha^2 R}{\partial t} + F^*\frac{\partial R}{\partial t} + F^2\ J(R,\nabla^2\alpha) + F^2\ \nabla^2\ J(\alpha,R) + \right.$$

$$\left. + F\ \beta\,\frac{\partial R}{\partial x}\right] \delta\alpha\ ds\ dt\ dp = 0 \tag{4.4}$$

$$\iint\limits_{SP} \left[(F\ \nabla^2\ R + F^* R)\ \delta\alpha \right]_0^T ds\ dp = 0 \tag{4.5}$$

$$\oint\!\!\int\limits_{LP} \left[F R\ \delta\ \frac{\partial \alpha}{\partial n} \right]_0^T dL\ dp = 0 \tag{4.6}$$

$$\oint\!\!\int\limits_{LP} \left[F\,\frac{\partial R}{\partial n}\delta d \right]_0^T dL\ dp = 0 \tag{4.7}$$

$$\oint\!\!\iint\limits_{LTP} \left\{ \left[(F\,\frac{\partial^2 R}{\partial t \partial n} - F^2\ \frac{\partial J(R,\alpha)}{\partial n}) \right] dL + \right.$$

$$\left. + \left[F^2 R\ \nabla(\nabla^2\alpha) + F R \nabla f \right] dL \right\} \delta\alpha\ dt\ dp = 0 \tag{4.8}$$

$$\oint\!\!\iint\limits_{LTP} \left[-F\,\frac{\partial R}{\partial t} + F^2 J(R,\alpha) \right] \delta\frac{\partial \alpha}{\partial n}\ dL\ dt\ dp = 0 \tag{4.9}$$

$$\oint\!\!\iint\limits_{LTP} F^2 R\ \nabla\alpha\ \delta\ \nabla^2\ \alpha\ dL\ dt\ dp = 0 \tag{4.10}$$

The main problem with regard to equations (4.4) - (4.10) is to decide if they are exactly satisfied by restrictions on α or its derivatives or if they give so called natural boundary or initial conditions, that have to be satisfied in order for R to be a minimum.

Since $\delta\alpha$ is arbitrary in S, T and P a necessary condition for equation (4.4) to be satisfied, directly gives the following forecast equation for R

$$\int_P \left\{ F\left(\frac{\partial \nabla^2 R}{\partial t} + \beta \frac{\partial R}{\partial x}\right) + F^2 \left[J(R, \nabla^2 \alpha) + \right.\right.$$
$$\left.\left. + \nabla^2 J(\alpha, R) \right] + F^* \frac{\partial R}{\partial t} \right\} dp = 0 \tag{4.11}$$

Taking (3.5) into account, we see that this in reality is a predictive equation for α of second order in time and of fourth order in S. Thus, in order to carry out a time integration using (4.10) we have to know α and $\partial\alpha/\partial t$ initially. On the other hand, since we wish to apply the equation to all kinds of initial fields, we do not wish to fix α at $T = 0$ once and for all and this means that $\delta\alpha$ may at this instant be arbitrary. In order for (4.5) to hold we must then have

$$\int_P \left[F \nabla^2 R + F^* R \right] dp = 0 \qquad t = 0 \tag{4.12}$$

which again taking (3.5) into account, determines the initial $\partial\alpha/\partial t$ as soon as initial α is given, provided that, at this instant we also have sufficient boundary conditions along L.

We shall here choose the simpliest possible case and assume the area S to be so large that we can have $\alpha = 0$ and $\partial\alpha/\partial n$ specified along L at all times. This makes $\delta\alpha = \delta\,\partial\alpha/\partial n = 0$ in equations (4.6) - (4.9) which thereby are satisfied. Furthermore condition (4.10) is also satisfied since $\nabla\alpha \cdot d\,L = d\alpha$ taken along L which vanishes with $\alpha = 0$. The variation $\delta\nabla^2\alpha$ along L is thus free.

5. Variation of F

In order to simplify notations when carrying out the second variation of (4.1) we introduce

$$A = \frac{\partial\nabla^2\alpha}{\partial t} + \beta\frac{\partial\alpha}{\partial x} \qquad B = J(\alpha,\nabla^2\alpha) \qquad C = f_0^2\frac{\partial\alpha}{\partial t} \tag{5.1}$$

and have thus to consider

$$\iiint\limits_{STP} R\,\delta\left[AF + BF^2 + C\frac{d}{dp}\left(\frac{1}{\sigma_0}\frac{dF}{dp}\right)\right] ds\,dt\,dp = 0 \tag{5.2}$$

For the third term inside the brackets we obtain after partial integration

$$\iiint\limits_{STP} RC\,\delta\frac{d}{dp}\left(\frac{1}{\sigma_0}\frac{dF}{dp}\right) ds\,dt\,dp =$$

$$\iint\limits_{ST} C\left[\frac{R}{\sigma_0}\delta\frac{dF}{dp} - \frac{1}{\sigma_0}\frac{\partial R}{\partial p}\delta F\right]_0^{p_s} ds\,dt + \tag{5.3}$$

$$+ \iiint\limits_{STP} C\frac{\partial}{\partial p}\left(\frac{1}{\sigma_0}\frac{\partial R}{\partial p}\right)\delta F\,ds\,dt\,dp$$

where, in the first term on the right hand side the integrand will vanish at $p = 0$ provided R and $\partial R/\partial p$ are finite. At the boundary $p = p_S$ we have from (3.8)

the relation

$$S_0 \delta \frac{dF}{dp} = \sigma_0 \delta F$$

so that the natural boundary condition on R, taking F unspecified and $\delta F = 0$ at $p = p_S$ becomes

$$\iint_{S\,T} C \left[\frac{R\sigma_0}{S_0} - \frac{\partial R}{\partial p}\right] ds\, dt = 0 \qquad (5.3)$$

Carrying out remaining variations in (5.2) and taking (5.3) into account we obtain

$$\iint_{S\,T} \left[R(A+2BF) + C \frac{\partial}{\partial p}\left(\frac{1}{\sigma_0} \frac{\partial R}{\partial p}\right)\right] ds\, dt = 0$$

which in view of the definition of R is a fourth order non-linear differential equation for F. Denoting

$$\iint_{S\,T} a(x,y,t)\, b(x,y,t)\, ds\, dt = \overline{ab}$$

and introducing R from (3.5) equations (5.4) and (5.3) become

$$\frac{d}{dp} \frac{1}{\sigma_0} \frac{d}{dp} \left[\overline{AC}\, F + \overline{BC}\, F^2 + \overline{C^2} \frac{d}{dp}\left(\frac{1}{\sigma_0} \frac{dF}{dp}\right)\right] +$$

$$\overline{AC} \frac{d}{dp}\left(\frac{1}{\sigma_0} \frac{dF}{dp}\right) + 2\,\overline{BC}\, F \frac{d}{dp}\left(\frac{1}{\sigma_0} \frac{dF}{dp}\right) + \qquad (5.5)$$

$$+ \overline{A^2} F + 3\,\overline{AB} F^2 + 2\,\overline{B^2} F^3 = 0$$

and at $p = p_S$

$$\frac{\sigma_0}{S_0}\left[\overline{AC}\,F + \overline{BC}\,F^2 + \overline{C^2}\,\frac{d}{dp}\left(\frac{1}{\sigma_0}\,\frac{dF}{dp}\right)\right] =$$

$$= \frac{d}{dp}\left[\overline{AC}\,F + \overline{BC}\,F^2 + \overline{C^2}\,\frac{d}{dp}\left(\frac{1}{\sigma_0}\,\frac{dF}{dp}\right)\right] \tag{5.6}$$

If we here divide by $\overline{C^2}$ and furthermore apply a normalizing condition on F, only four statistical quantities need to be known in order to solve equation (5.5) with the boundary conditions (5.6) and (3.8). However, these statistics are not known.

The situation is in some respects similar to the one in linear problems. An infinity of solutions exist, all satisfying the equation and the boundary conditions. And in order to make the solution determinate we need to know initial data and - if dissipation is included - also the external forcing.

Data of this kind are not directly available. The functions $\alpha(x,y,t)$ cannot be determined from observations unless F is given and vice versa. We therefore have to treat atmospheric data in a different way in order to find solutions to equation (5.5).

In the approximate relation (3.4) we shall now let $\psi(x,y,t,p)$ represent measured values of the stream function, derived from geopotential data by means of the geostrophic approximation. Utilizing these data we now wish to determine empirical functions $\alpha(x,y,t)$ and $F(p)$ with the condition that the residual in the approximation becomes as small as possible. The variational problem is

$$\delta \iiint_{STP} \left[\psi(x,y,t,p) - \alpha(x,y,t)\,F(p)\right]^2 ds\,dt\,dp = 0$$

where both α and F are to be varied. It is not sur-

prising that we here recognize the basic principle in the derivation of empirical orthogonal functions - in too many applications thought of only as eigenvectors to a covariance matrix. Carrying out the variation we obtain a system of two integral equations from which either α or F can be eliminated. Chosing to eliminate α we find

$$\mu\int K(p,p')F(p')dp' = F(p) \qquad (5.7)$$

or in matrix notation

$$KF = \lambda F \qquad \lambda = 1/\mu$$

where the kernel $K(p,p')$ is the covariance function (matrix)

$$K(p,p') = \iint\limits_{ST} \psi(x,y,t,p)\psi(x,y,t,p')ds\,dt$$

and μ, the eigenvalue of the Fredholm integral equation is defined by

$$\frac{1}{\mu} = \iint\limits_{ST} \alpha^2(x,y,t)ds\ dt \cdot \int\limits_{P} F^2(p)dp$$

The eigenfunctions of (5.7) form an orthogonal set and give in a series expansion of observed stream function data the fastest possible convergence with an optimized variance reduction for each term.

It is now of interest to discuss the extent to which our theoretically derived functions $\alpha(x,y,t)$ and $F(p)$ can be identified with those obtained from expansions of observational data. Going back to equation (3.3) we may there introduce $\psi = \alpha F + \psi'$ and move to the right hand

side all terms that contain ψ', retaining on the left hand side only those that are found in the same place in (3.5). Taking then $\psi' = 0$ at each initial moment, which is necessary since this is the only approximation we permit, we see that R is nothing but the instantaneous production of potential vorticity in ψ' and that R^2 is the corresponding creation of potential enstrophy. But this we have minimized and equation (5.5) will therefore give the vertical structure of modes in the atmosphere that for as long as possible will retain their enstrophy, the loss being due to the interaction of the mode with itself. Provided these modes are generated by the external forcing one would also expect them to be dominant in the atmosphere and it should therefore be possible to find solutions to (5.5) that are identical or almost identical to those found from empirical data.

One major condition for this is, however, essential. The equation for F is derived from a combination of the vorticity and the adiabatic equations in a form where a considerable amount of approximation has been made and where for instance friction has been entirely neglected. A complete identity is therefore not to be expected. Instead it would seem posssible to take advantage of deviations in order to detect approximations that should not have been made.

6. Linearized model

In the previous sections we have used the stream function as dependent atmospheric variable. Since no expansion into empirical orthogonal functions exist on such data we shall here compare with results from expansions of observed geopotential heights, taking advantage of the geostrophic approximation. A certain caution is here necessary since the approximation is not always valid.

Expansions of geopotential data have been made by Obukhov (1960), Holmström (1963) and others and we shall here, for reference, utilize results obtained by Holmström. The first four empirical functions $F(p)$

are reproduced in fig. 1. They are normalized to 1 over the interval 1000 - 100 mb. The dominance of the first mode in these hemispheric data from 7 days in October 1959 is very pronounced with r.m.s. values of corresponding α - functions being 236.2, 42.5, 20.1 and 10.2 m. respectively.

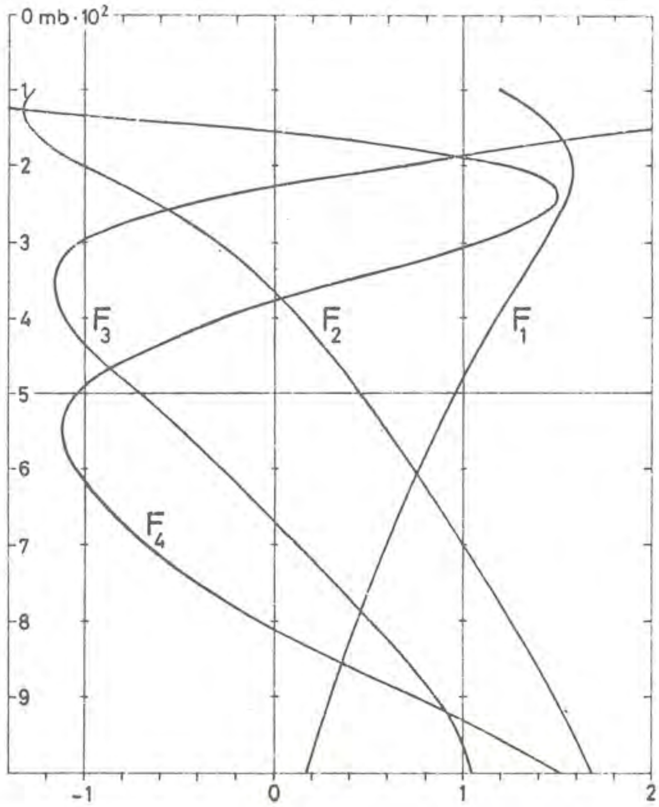


Fig. 1 First four functions F(p) in expansion of geopotential data into empirical orthogonal functions.

For the comparison between theoretical and empirical results we shall not in this paper use the full equation (5.5) but instead consider a simplified version which is more easily discussed. Since the circulation pattern of the atmosphere to a major part is characterized by waves of rather sinusoidal form and relatively small amplitude it seems reasonable to assume that equation (5.5) would give realistic vertical profile functions even if the corresponding α - field is linearized. We shall therefore here only consider small perturbations on a zonal flow and for α introduce

$$\alpha = -U_0 y + \hat{\alpha} e^{ik(x-ct)} \tag{6.1}$$

where U_0 and $\hat{\alpha}$ are constants and $\hat{\alpha}$ is small. Instead of introducing this expression in (5.5) we may here proceed in a different way. From (3.5) and (6.1) we obtain

$$R = \left[(k^2 c + \beta) F - k^2 U_0 F^2 - cF^*\right] ik\hat{\alpha} e^{ik(x-ct)}$$

and from (4.10)

$$\int \left[(k^2 c + \beta) F - k^2 U_0 F^2 - cF^*\right]^2 dp = 0$$

where second order quantities in $\hat{\alpha}$ have been neglected. If we here look for stable solutions with the phase velocity c being real, this relation cannot hold unless the integrand vanishes at all p. We therefore have

$$\frac{d}{dp}\left(\frac{1}{\sigma_0}\frac{dF}{dp}\right) = \frac{k^2 c + \beta}{f_0^2 c} F - \frac{k^2 U_0}{f_0^2 c} F^2 \tag{6.2}$$

where the expression for F^* in (4.2) has been reintroduced.

A few characteristic properties of solutions to eq. (6.2) were discussed by Holmström (1964) but there the equation was not properly derived and the presentation of computational results not adequate. An extended treatment is therefore given here.

Equation (6.2) which for σ_0 = constant has elliptic functions as solutions, has with σ variable also certain interesting properties. First of all it is directly seen that if $F = 1$ we obtain the Rossby phase velocity relation. A slightly modified form is also obtained if the equation is integrated over p

with the boundary conditions $dF/dp = 0$ at 1000 mb and $dF/\sigma_0 dp = 0$ at $p = 0$ the modification depending on the integrated value of the normalized function F. By taking $F = \mu G$ where μ is a constant it is also easily shown that the coefficient of F^2 has no influence on the form of the solution. It determines only its amplitude and sign and may therefore be arbitrarily chosen so as to make the solution normalized and to be positive at $p = 1000$ mb. As will be seen the properties of the solution are only determined by its absolute value at 1000 mb and by the cofficient in (6.2) multiplying F. This coefficient has here the character of an eigenvalue but the similarity with linear equations is not complete. It is convenient for the demonstration to introduce the notation

$$a^2 = \frac{k^2 c + \beta}{f_0^2 c} \quad i \quad b^2 = \frac{k^2 U_0}{f_0^2 c} \tag{6.3}$$

so that the equation becomes

$$\frac{d}{dp}\left(\frac{1}{\sigma_0}\frac{dF}{dp}\right) = a^2 F - b^2 F^2 \tag{6.4}$$

If we here introduce

$$F = \mu G + \nu \tag{6.5}$$

where μ and ν are constants, we find from (6.4)

$$\mu\frac{d}{dp}\left(\frac{1}{\sigma_0}\frac{dG}{dp}\right) = (a^2-2b^2\nu)\mu G + \nu(a^2-\nu b^2)-b^2\mu^2 G^2 \tag{6.6}$$

If we here determine ν so that

$$a^2 - \nu b^2 = 0 \tag{6.7}$$

equation (6.6) transforms into

$$\frac{d}{dp}\left(\frac{1}{\sigma_0}\frac{dG}{dp}\right) = -a^2 G - \mu b^2 G^2 \tag{6.8}$$

which only differs from (6.4) by the sign of the eigenvalue and by possibly a different normalisation of G depending on the arbitrary constant μ.

Multiplying now (6.8) by F, (6.4) by G and taking the difference we obtain

$$\frac{d}{dp}\left[\frac{G}{\sigma_0}\frac{dF}{dp} - \frac{F}{\sigma_0}\frac{dG}{dp}\right] = 2a^2 FG - b^2 FG(F-\mu G) = a^2 FG$$

where the relations (6.5) and (6.7) have been taken into account. It is here seen that F and G are orthogonal functions provided they satisfy a boundary condition of the type given in (3.8). On this condition we have to (6.4) pairs of mutually orthogonal solutions corresponding to opposite signs of the eigenvalue and related linearly through equation (6.5).

With this result in mind it is interesting to compare the two first functions given in fig. 1. Determining μ and ν in (6.5) so as to give the best possible fit in the intervals 1000 - 100 mb and 1000 - 250 mb we obtain from values of F_1 the curves F_2' and F_2'' in fig. 2, compared with F_2. In spite of the difference above 250 mb the similarity between F_2 and F_2'' is striking and seems to imply the validity of this characteristic also in empirical data.

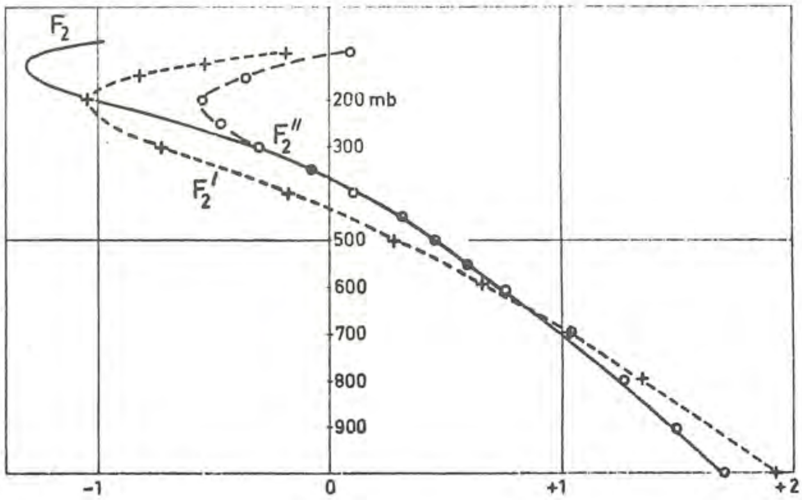


Fig. 2 Comparison between the function $F_2(p)$ of fig. 1 with corresponding functions $F_2'(p)$ and $F_2''(p)$ calculated from $F_1(p)$.

A further point of interest in equation (6.4) is the role of the non-linear term. Taking a^2 positive the effect of the linear term is to make the solution diverge from the p-axis. For positive F the non-linear term with its negative sign is therefore necessary in order to recurve the solution towards zero and to satisfy realistic boundary conditions. For negative F the solution is seen to diverge and consequently no boundary condition at p=0 can be satisfied.

Turning now to numerical integrations of equation (6.4) a sample of calculated profiles for F is shown in figure 3. Integrations have been carried out from prescribed initial values of F at 1000 mb in steps of 1 mb and in the first step satisfying the boundary condition (3.8). Values for σ_0 have been taken from Holmström (1963), interpolated in the interval 1000 - 200 mb and from there on extrapolated assuming a smooth transition to an isothermal stratosphere. The solutions have not been renormalized.

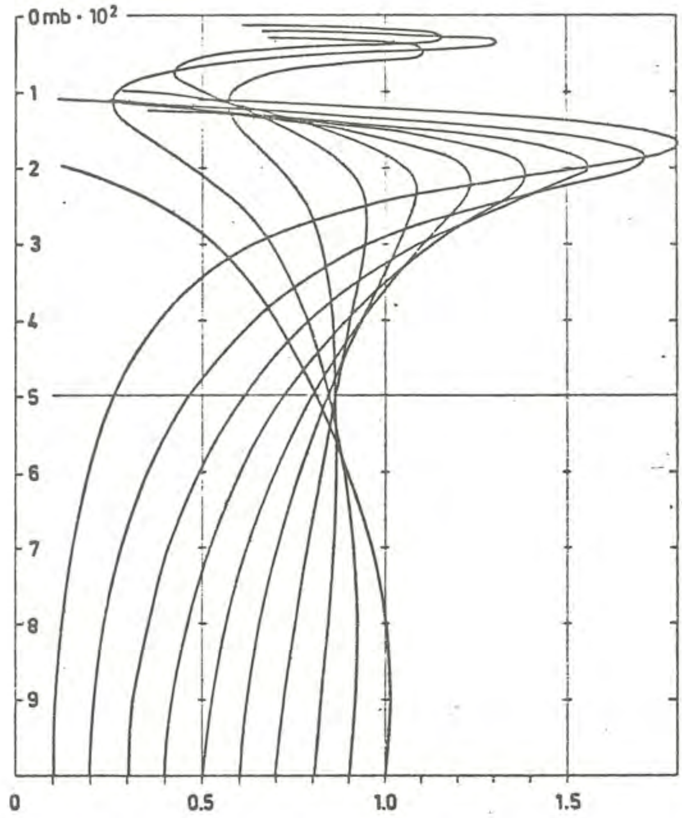


Fig. 3 Profiles F(p) calculated from eq. (6.4) with $a^2 = 5 \cdot 10^{-4}$, $b^2 = 6.25 \cdot 10^{-4}$ and varying values of F at p = 1000 mb.

Calculations in figure 3 have been made with $a^2 = 5 \cdot 10^{-4}$ and $b^2 = 6.25 \cdot 10^{-4}$. Taking $f_0 = 10^{-4}$ and $\beta = 10^{-11}$ various values of k(or wavelength L) correspond to the values of C and U_0 given in fig. 4. Different initial values at p = 1000 mb have been prescribed and we shall here use these for reference.

From fig. 3 it is seen that all curves become negative before reaching 5 mb. They will diverge rapidly towards infinite values and the curves therefore in this cal-

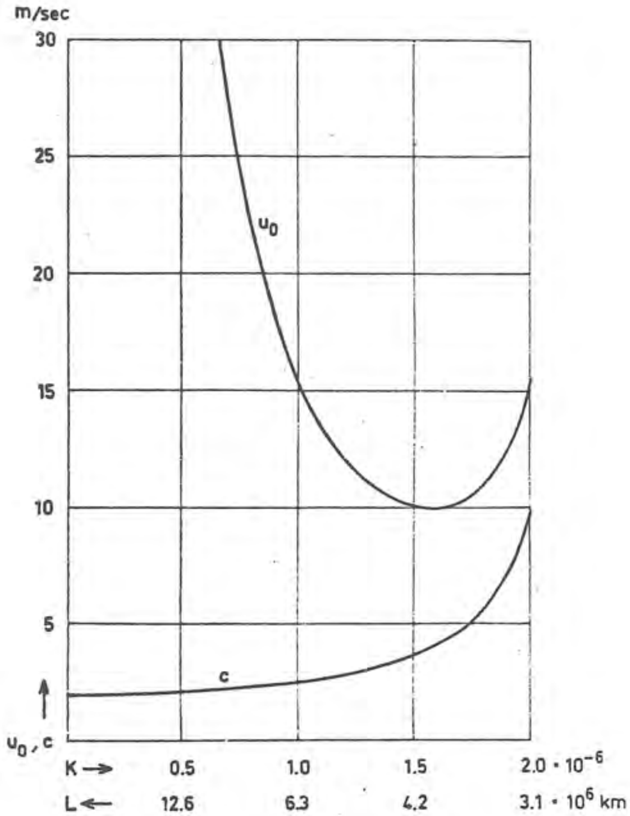


Fig. 4 Relation between wave number K and phase velocity c and zonal wind u_o for values of a^2 and b^2 used in fig. 3.

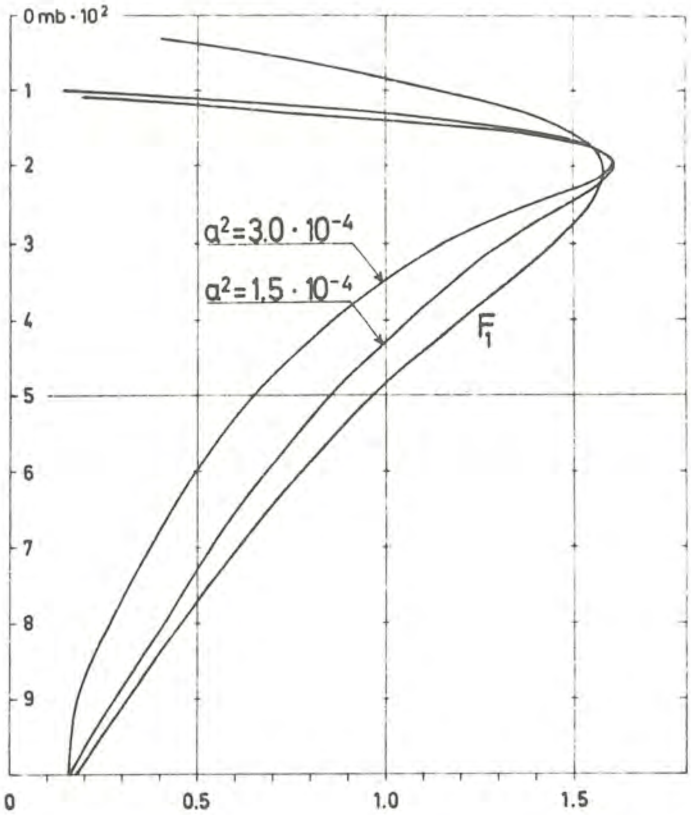
culation do not represent solutions that satisfy the upper boundary condition. In some cases this may be due to the integration step. 1 mb, being too large close to the singularity, but this point is here not of special interest. More interesting is the fact that vertical gradients as well as curvature become very large even in a z-system and one would therefore expect turbulence to be generated at certain levels. The neglect of friction in the initial set of equations is therefore likely to be of crucial importance.

The effect of the neglect of friction is also evident in the curves 0.6 - 1.0, which give unrealistic values of the wind at the ground, taking the geostrophic approximation into account. The drag should lower considerably the value of the function at 1000 mb. Naturally a re-

derivation of the equations should therefore be made, including from the beginning an expression for the friction. This would also affect the geostrophic approximation and make for instance the Jacobian in the thermodynamic equation non-vanishing. It would however also necessitate inclusion of an external forcing to balance dissipation and this would not only complicate the treatment but also make the results dependent on rather arbitrary assumptions. We shall therefore in this paper take surface friction into account simply by relaxing the lower boundary condition (3.8) and carry out the calculations and the comparison assuming that at 1000 mb the theoretical solution should have the same value and the same slope as the empirical function.

Recomputing F with this boundary condition we obtain the results shown in fig. 5 for varying values of a^2 and b^2. Up to levels around 250 mb some of the curves show a striking similarity with the empirical one but above this level we again notice a much larger curvature and much steeper gradients. This may be due to lack of friction but other approximations may also be involved.

We also notice that solutions remain rather similar for a wide range of values of a^2. This implies that the vertical structure of the model atmosphere and most probably also the real atmosphere does not to a large extent depend on the circulation pattern itself but much more on the average static stability and on surface friction. This is corroborated by the dominance of the first empirical orthogonal function in expansions of the geopotential. One may here also see a reason why one parameter models indeed give comparatively good results in short term forecasting.



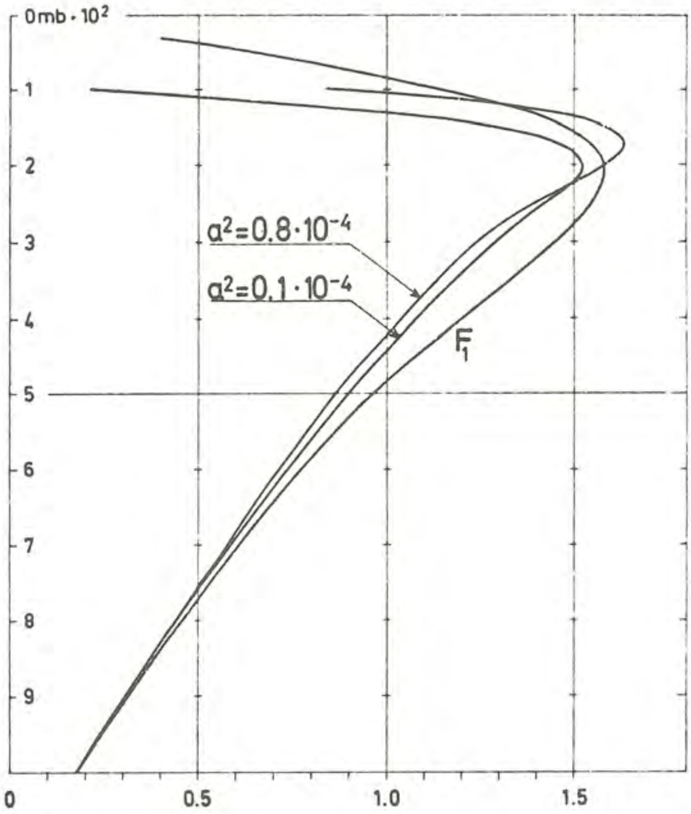


Fig. 5a,b Profiles F(p) calculated from eq. (6.4) with relaxed boundary condition at p = 1000 mb and compared with empirical $F_1(p)$. Values of a^2 are given beside the curves.

7. Conclusion

The purpose in this paper has been to draw attention to the fact that in parameterizing a non-linear predictive equation - as we always do in meteorology - the parameters should not be predicted with the same equation as the variable they replace. Instead a higher order equation may be derived which minimizes the approximation made when introducing the parameters. The variational method employed also defines eigensolutions to the mathematical model of the atmosphere. Some of these should be similar to those found from empirical data. If this condition is not satisfied the conclusion should be that the mathematical model does not sufficiently well simulate the real atmosphere. In such a case one also cannot expect the predictive equations to give useful results.

Starting from a set of approximative equations and drastically simplifying computations we have shown that the eigenfunctions to the mathematical model have certain characteristic properties common with the real atmosphere. However, certain obvious differences exist. These may be due to neglect of friction in the mathematical model but other sources of error are also possible. A more comprehensive treatment of the problem is therefore required in order to determine the simplest possible mathematical model of the atmosphere which still has "eigenfunctions" that sufficiently well resemble the lowest three or four empirical functions.

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Appendix A

Starting with the first term in (4.2) we first integrate partially with respect to time, obtaining

$$\iiint\limits_{STP} R\,F\,\delta\frac{\partial\nabla^2\alpha}{\partial t}\,ds\,dt\,dp = \iint\limits_{SP}\Big[R\,F\,\delta\nabla^2\alpha\Big]_0^T ds\;dp - \tag{A.1}$$

$$- \iiint\limits_{STP} F\frac{\partial R}{\partial t}\,\delta\nabla^2\alpha\,ds\,dt\,dp$$

since F is a function of p only. On both terms on the right hand side we now utilize Greens theorem

$$\int\limits_S a\,\nabla^2 b\,ds = \int\limits_S b\,\nabla^2 a + \oint\limits_L\left(a\frac{\partial b}{\partial n} - b\frac{\partial a}{\partial n}\right) dL \tag{A.2}$$

This gives directly

$$\left.\begin{aligned}
&\iiint\limits_{STP} R\,F\,\delta\frac{\partial\nabla^2\alpha}{\partial t}\,ds\,dt\,dp = \\
&= \iint\limits_{SP}\Big[F\,\nabla^2 R\,\delta\,\alpha\Big]_0^T + \oint\!\!\int\limits_{LP} F\Big[R\,\delta\frac{\partial\alpha}{\partial n} - \frac{\partial R}{\partial n}\,\delta\,\alpha\Big]_0^T dL\,dp - \\
&- \iiint\limits_{STP} F\frac{\partial\nabla^2 R}{\partial t}\,\delta\,\alpha\,ds\,dt\,dp - \oint\!\!\iint\limits_{LTP} F\,\Big(\frac{\partial R}{\partial t}\,\delta\frac{\partial\alpha}{\partial n} - \\
&- \frac{\partial^2 R}{\partial t\,\partial n}\,\delta\,\alpha\Big)\,dL\,dt\,dp
\end{aligned}\right\} \tag{A.3}$$

The next term to be dealt with is the Jacobian, where we first notice that, since δ is a linearizing operator, we have

$$\iiint\limits_{STP} R F^2 \delta\, J(\alpha, \nabla^2\alpha)\, ds\, dt\, dp = \iiint\limits_{STP} R F^2 \Big[J(\delta\alpha, \nabla^2\alpha) - \tag{A.4}$$

$$- J(\delta\, \nabla^2\, \alpha,\, \alpha) \Big] ds\, dt\, dp$$

We shall also utilize the following two relations, which are easily verified

$$J(a,b) = \mathbb{k} \cdot (\nabla a \times \nabla b) = \mathbb{k} \cdot \left[\nabla \times (a \nabla b) \right] \tag{A.5}$$

and, if $c = c(x,y)$

$$c\, J(a,b) = J(ac,b) - a\, J(c,b) \tag{A.6}$$

Combining (A.5) and (A.6) we have

$$\int\limits_S c\, J(a,b)\, ds = \int\limits_S J(ac,b)\, ds - \int\limits_S a\, J(c,b)\, ds =$$

$$= \int\limits_S \left[\nabla \times (ac\, \nabla\, b) \right] d\mathbb{s} - \int a\, J(c,b)\, ds$$

or, using Stokes theorem

$$\int\limits_S c\, J(a,b)\, ds = \int\limits_L ac\, \nabla\, b \cdot d\mathbb{L} - \int\limits_S a\, J(c,b)\, ds \tag{A.7}$$

In these equations $d\mathbb{s}$ is a vector surface element of S and $d\mathbb{L}$ a vector line element along L.

Combining now (A.4) and (A.7) and taking $c = R$, $a = \delta a$ and $b = \nabla^2\alpha$ in the first case and $a = \delta\nabla^2\alpha$ and $b = \nabla^2\alpha$ in the second, we find

$$\iiint\limits_{STP} R F^2 \delta\, J(\alpha, \nabla^2\alpha)\, ds\, dt\, dp =$$

$$\oint\!\!\int\!\!\int_{LTP} F^2 R \nabla(\nabla^2\alpha)\,\delta\alpha\cdot d\mathbb{L}\,dt\,dp - \int\!\!\int\!\!\int_{STP} F^2 J(R,\nabla^2\alpha)\,\delta\alpha\,ds\,dt\,dp -$$

$$- \oint\!\!\int\!\!\int_{LTP} F^2 R \nabla\alpha\cdot\delta\nabla^2\alpha\,d\mathbb{L}\,dt\,dp + \int\!\!\int\!\!\int_{STP} F^2 J(R,\alpha)\,\delta\nabla^2\alpha\,ds\,dt\,dp$$

or, using (A.2) on the last integral

$$\left.\begin{aligned}
&\int\!\!\int\!\!\int_{STP} R F^2 \delta J(\alpha,\nabla^2\alpha)\,ds\,dt\,dp = \\
&- \int\!\!\int\!\!\int_{STP} F^2 \left[J(R,\nabla^2\alpha) + \nabla^2 J(\alpha,R)\right]\delta\alpha\,ds\,dt\,dp \\
&+ \oint\!\!\int\!\!\int_{LTP} F^2 R\left[\nabla(\nabla^2\alpha)\,\delta\alpha - \nabla\alpha\delta\nabla^2\alpha\right]\cdot d\mathbb{L}\,dt\,dp \\
&+ \oint\!\!\int\!\!\int_{LTP} F^2\left[J(R,\alpha)\frac{\partial\delta\alpha}{\partial n} - \frac{\partial J(R,\alpha)}{\partial n}\,\delta\alpha\right]dl\,dt\,dp
\end{aligned}\right\} \quad (A.8)$$

The variation of the β-term is easiest to carry out if we write it under the form of a Jacobian. Repeating previous procedures we find

$$\left.\begin{aligned}
&\int\!\!\int\!\!\int_{STP} \beta F R \delta\frac{\partial\alpha}{\partial x}\,ds\,dt\,dp = \int\!\!\int\!\!\int_{STP} F\Big[J(R\delta\alpha,f) - \\
&- J(R,f)\,\delta\alpha\Big]\,ds\,dt\,dp \\
&= \oint\!\!\int\!\!\int_{LTP} F R \nabla f\,\delta\psi\cdot d\mathbb{L}\,dt\,dp - \int\!\!\int\!\!\int_{STP} F\beta\frac{\partial R}{\partial x}\,\delta\alpha\,ds\,dt\,dp
\end{aligned}\right\} (A.9)$$

Collecting now results and including also the transformation of the time-derivative from the adiabatic equation we find

$$
\left.
\begin{aligned}
& - \iiint\limits_{S\,T\,P} \left[F \frac{\partial \nabla^2 R}{\partial t} + F^* \frac{\partial R}{\partial t} + F^2 J(R, \nabla^2 \alpha) + \right. \\
& \left. + F^2 \nabla^2 J(\alpha, R) + F \beta \frac{\partial R}{\partial x} \right] \delta\alpha \, ds\, dt\, dp \\
& + \iint\limits_{S\,P} \Big[(F \nabla^2 R + F^* R)\, \delta\alpha \Big]_0^T ds\, dp + \\
& + \oint\!\!\int\limits_{L\,P} F \left[R\, \delta \frac{\partial \alpha}{\partial n} - \frac{\partial R}{\partial n} \delta\alpha \right]_0^T dL\, dp \\
& + \oint\!\!\iint\limits_{L\,T\,P} \{ F \left(\frac{\partial^2 R}{\partial t \partial n} \delta\alpha - \frac{\partial R}{\partial t} \delta \frac{\partial \alpha}{\partial n} \right) + \\
& + F^2 \left[J(R,\alpha)\, \delta \frac{\partial \alpha}{\partial n} - \frac{\partial J(R,\alpha)}{\partial n} \delta\alpha \right] \} \, dL\, dt\, dp \\
& + \oint\!\!\iint\limits_{L\,T\,P} \{ F^2 R \left[\nabla (\nabla^2 \alpha)\, \delta\alpha - \nabla\alpha\, \delta \nabla^2 \alpha \right] + \\
& + F R \nabla f\, \delta\alpha \} \cdot d\mathbb{L}\, dt\, dp = 0
\end{aligned}
\right\} \qquad (A.10)
$$

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